

Topic 9.4 Hyperbolas

Essential Question:

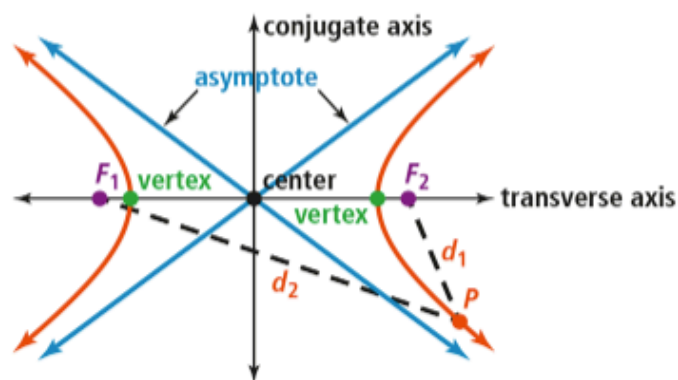
How does the equation of a hyperbola relate to the features of its graph?

CONCEPT: Hyperbolas

A **Hyperbola** is the set of all points P such that the difference of the distances from any point P to two fixed points, or **foci** (singular: focus) **of a hyperbola**, is constant. A hyperbola has two branches and two **asymptotes**.

A line drawn through the foci intersects the hyperbola at two points called the **vertices of a hyperbola**.

The **foci** and the **vertices** lie along the **transverse axis**. The **conjugate axis** is perpendicular to the transverse axis and passes through the **center of a hyperbola**.



A hyperbola is the intersection of a plane with an infinite double right cone such that the plane intersects both of the cones.

Q: How are the two branches of the hyperbola related to the conjugate axis?

Q: How does the graph of a hyperbola change if the transverse axis is vertical?

Notes:

CONCEPT Summary

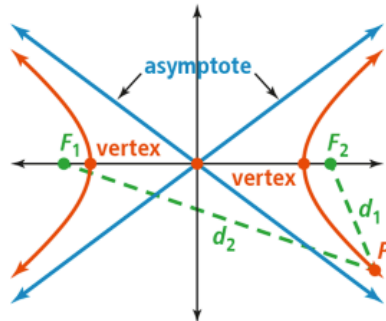
Hyperbolas

DEFINITION

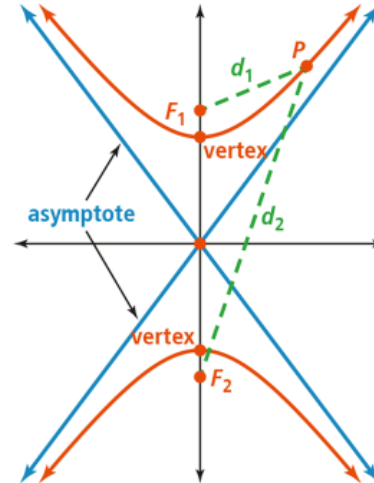
A hyperbola is the set of all points P such that the difference of the distances from any point P to two fixed points is constant.

GRAPHS

Horizontal Hyperbola



Vertical Hyperbola



EQUATION

Horizontal: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

vertices: $(\pm a, 0)$

asymptotes: $y = \pm \frac{b}{a}x$

Foci: $(\pm c, 0)$, where $c = \sqrt{a^2 + b^2}$

Vertical: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

vertices: $(0, \pm a)$

asymptotes: $y = \pm \frac{a}{b}x$

foci: $(0, \pm c)$, where $c = \sqrt{a^2 + b^2}$

Q: What is the relationship between d_1 , and d_2 in the graph of the hyperbola?

Notes:

Examples & Questions

Examples 1

Q: Why is absolute value used with the expression $d_2 - d_1$?

Q: How is deriving the equation of a hyperbola different from deriving the equation of an ellipse?

Examples 2

Q: How are the asymptotes of a hyperbola different from asymptotes of other functions you have studied?

Q: How does the rectangle you drew relate to the asymptotes of the hyperbola?

Q: Is this relationship true for all hyperbolas? Why or why not?

Q: What relationship do you see between the foci and the vertices on the graph of a hyperbola?

Examples 3

Q: How can you determine if the transverse axis is the x -axis or y -axis, given the ordered pairs of the vertices?

Q: When would you use the equation of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ versus $\frac{y^2}{a^2} - \frac{x^2}{b^2} = -1$.

Examples 4

Q: What method would you use to find the distance that the detector should be placed from the mirror?

Q: Why do you consider only one branch of the hyperbola in this situation?

Examples 5

Q: Why is a second-degree equation a circle when $A = C$?

Q: Can the absolute values of A and C be equal but the equation not represent a circle? Explain.

Practice and Problem Solving

Complete MathXL for School: Practice and Problem Solving (online)

Complete MathXL for School: Enrichment (online)

Challenge: #36– key will be posted in Power School Learning.

Lesson Quiz 9.4