Topic 5.2 Properties of Exponents and Radicals

Essential Question:

How can you solve rational equations and identify extraneous solutions?

CONCEPT

Properties of Rational Exponents

The properties of exponents apply not only to integer exponents, but to *rational* exponents as well. Now let *m* and *n* represent *rational* numbers, with *a*, *b* nonnegative real numbers.

	Property	Example
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$4^{\frac{2}{3}} \bullet 4^{-\frac{1}{3}} = 4^{\frac{1}{3}}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^4}{3^2} = 3^{4-2} = 3^2 = 9$
Power of Power	$(a^m)^n = a^{mn}$	$(7^3)^{\frac{2}{3}} = 7^2$
Power of Product	$(ab)^m = a^m b^m$	$(16x)^{\frac{1}{2}} = (16^{\frac{1}{2}}x^{\frac{1}{2}}) = 4x^{\frac{1}{2}}$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$	$5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}}$

Q: What are some things that you notice about the properties of rational exponents?

Notes

CONCEPT Summary

Properties of Radicals

	Product Property of Radicals	Quotient Property of Radicals	Rationalize the Denominator
WORDS	The *n*th root of a product is equal to the product of the *n*th roots of the factors.	The *n*th root of a quotient is equal to the quotient of the *n*th roots of the factors.	To rationalize the denominator of an expression, multiply by the conjugate of the denominator.
ALGEBRA	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\frac{3}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3\sqrt{x}}{x}$
NUMBERS	$\sqrt[3]{2} \cdot \sqrt[3]{20} = \sqrt[3]{40}$ $\sqrt[3]{40} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$	$\sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}} = \frac{2\sqrt{2}}{3}$	$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Using Properties of Radicals

SIMPLIFY

$$\sqrt[4]{32x^9} = \sqrt[4]{16x^8} \cdot \sqrt[4]{2x}$$
$$= 2x^2\sqrt[4]{2x}$$

Find factors that have a perfect 4th root.

$$\frac{5}{x-\sqrt{8}} \cdot \frac{x+\sqrt{8}}{x+\sqrt{8}} = \frac{5(x+\sqrt{8})}{x^2 + x\sqrt{8} - x\sqrt{8} - 8} = \frac{5x+5\sqrt{8}}{x^2-8}$$

Since the denominator is a binomial, multiply the numerator and the denominator by the conjugate of the denominator.

Q: How are the properties of radicals similar to the properties of exponents?

Examples & Questions

Examples 1

Part A

Q: What property of rational exponents was used?

Part B

Q: What property of rational exponents was used?

Q: What would happen if you used the Power of Power Property first?

Examples 2

Q: How do you rewrite a radical as a rational exponent?

Q: What is your first step when simplifying a radical?

Examples 3

Q: Why is it necessary to rewrite the expression using rational exponents in Part B, but not in Part A?

Q: Why is the factor $\frac{\sqrt[3]{3m^2}}{\sqrt[3]{3m^2}}$ used in Part C?

Examples 4

Q: Can radical terms be combined if they have the same index but different radicands?

Q: How are like radicals similar to like terms? Can they be combined in the same way?

Examples 5

Part A

Q: Why is $\sqrt[3]{7}$ times $\sqrt[3]{49}$ equal to $\sqrt[3]{343}$ and not $\sqrt[9]{343}$?

Part B

Q: How could you expand the product of binomial factors with radicals?

Examples 6

Q: Why is it necessary to multiply a denominator that includes a binomial with a radical by its conjugate in order to rationalize, rather than simply multiplying by a single radical?

Practice and Problem Solving

Complete MathXL for School: Practice and Problem Solving (online)

Complete MathXL for School: Enrichment (online)

Challenge: # 55 – key will be posted in Power School Learning.

Lesson Quiz 5.2