Topic 5.1 nth Roots, Radicals and Rational Exponents

Essential Question:

How are exponents and radicals used to represent roots of real numbers?

CONCEPT

CONCEPT Interpreting Fractional Exponents

The index of a radical is equivalent to the denominator of a fractional exponent.

In general, if the nth root of c is a real number, $\sqrt[n]{c} = c^{\frac{1}{n}}$.

Furthermore, if m is an integer and $\frac{m}{n}$ is in lowest terms, then $c^{\frac{m}{n}} = \left(c^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{c}\right)^m$ and $\sqrt[n]{c^m} = \left(c^m\right)^{\frac{1}{n}} = c^{\frac{m}{n}}$.

CONCEPT

CONCEPT Solving an Equation in the Form $x^n = c$

To solve an equation in the form $x^n = c$, find the nth root of both sides by raising each expression to the $\frac{1}{n}$ power.

$$\left(x^n\right)^{\frac{1}{n}} = (c)^{\frac{1}{n}}$$

Notes

CONCEPT SUMMARY

CONCEPT SUMMARY *n*th Roots, Radicals, and Rational Exponents

Concept Summary · Assess

	Relating Radical and Exponential Forms	Solving an Equation in the Form $x^n = c$
WORDS	The index of a radical is equivalent to the denominator of a fractional exponent. The exponent of the radicand is equivalent to the numerator of a fractional exponent.	To solve an equation in the form $x^n = c$, find the *n*th root of both sides of the equation by raising each expression to the $\frac{1}{n}$ power.
NUMBERS	**Radical Form** $\sqrt[5]{32^4} = (32^4)^{\frac{1}{5}} = 32^{\frac{4}{5}}$ **Exponential Form** $729^{\frac{5}{6}} = \left(729^{\frac{1}{6}}\right)^5 = (\sqrt[6]{729})^5$	$x^3 = 1{,}728$ $(x^3)^{\frac{1}{3}} = (1{,}728)^{\frac{1}{3}}$ $x = 12$
ALGEBRA	**Radical Form** $\sqrt[n]{c^m} = (c^m)^{\frac{1}{n}} = c^{\frac{m}{n}}$ **Exponential Form** $c^{\frac{m}{n}} = \left(c^{\frac{1}{n}}\right)^m = \sqrt[n]{c^m}$	$x^n = c$ $(x^n)^{\frac{1}{n}} = (c)^{\frac{1}{3}}$ $x = c^{\frac{1}{n}}$

Q: How can you check that you have rewritten an expression in radical form as an expression in exponential form correctly?

Examples & Questions

Examples 1

Q: When finding the *n*th root of a number, what do you notice about the real and complex solutions?

Q: How are the real roots of an even power related?

Examples 2

Q: Does the order in which you raise a number to a power and find the *n*th root matter?

Q: What do you know about the value of *n* if *x* is negative?

Q: Could a rational exponent have a 1 in the denominator? What does that represent?

Examples 3

Q: Why might it be easier to calculate the *n*th root before raising to a power

Q: What is another method you could use to evaluate an expression that has a negative rational exponent?

Q: How is the Fujita scale classification related to the value of *F* you calculated?

Q: Why is a calculator a useful tool when approximating a cube root?

Examples 4

Part A:

Q: What is another approach you can use to simplify the expression?

Part B:

Q: Why is $|x^5|$ written as an absolute value expression?

Examples 5

Q: Why is the Power of a Product Property used when solving this equation?

Examples 6

Q: What does *volume* mean in the context of this problem?

Q: What do you notice about the image that is important to understanding and solving this problem?

Practice and Problem Solving

Complete MathXL for School: Practice and Problem Solving (online)

Complete MathXL for School: Enrichment (online)

Challenge: #50 – key will be posted in Power School Learning.

Lesson Quiz 5.1