

## Topic 4.2: Graphing Rational Functions

Essential Question:

*How are inverse variation and reciprocal functions related and represented?*

### Explore & Reason

Please complete online.

### CONCEPT

#### CONCEPT BOX Rational Functions

Just as a rational number is a number that can be expressed the ratio of two integers, a **rational expression** is an expression that can be expressed as the ratio of two polynomials, such as  $\frac{P(x)}{Q(x)}$ , where the value of  $Q(x) \neq 0$ .

A **rational function** is any function defined by a rational expression, such as  $R(x) = \frac{P(x)}{Q(x)}$ . The domain of  $R(x)$  is all values of  $x$  for which  $Q(x) \neq 0$ .

The function  $g(x) = \frac{4x}{x-3}$  is a rational function.

### Notes

# Concept Summary

Q: What are the key features of graphs of rational functions?

## CONCEPT SUMMARY Graphing Rational Functions

### RATIONAL FUNCTION

A function that is expressible as a fraction with polynomials in the numerator and the denominator

### ASYMPTOTES

#### Vertical

Vertical asymptotes are guides for the behavior of a graph as it approaches a vertical line.

- The line  $x = a$  is a vertical asymptote of  $\frac{P(x)}{Q(x)}$ , if  $Q(a) = 0$  and  $P(a) \neq 0$ .
- The up or down behavior of the function as it approaches the asymptote can be determined by substituting values close to  $a$  on either side of the asymptote.

#### Horizontal

Horizontal asymptotes are guides for the end behavior of a graph as it approaches a horizontal line.

- If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at  $y = 0$ .
- greater than the denominator, there is no horizontal asymptote.
- equal to the degree of the denominator, set  $y$  equal to the ratio of the leading coefficients. The graph of this line is the horizontal asymptote.

### ALGEBRA

$$f(x) = \frac{8x - 3}{4x + 1}$$

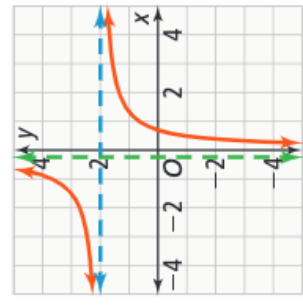
Vertical Asymptote: Let  $4x + 1 = 0$  and solve.

$$x = -\frac{1}{4}$$

Horizontal Asymptote: Find the ratio of the leading coefficients  $\left(\frac{8}{4}\right)$ .

$$y = 2$$

### GRAPH



## Examples & Questions

### Examples 1

Q: What do the asymptotes of  $g(x)$  look like compared to the parent function  $f(x) = \frac{1}{x}$ ?

### Examples 2

Part A:

Q: How do you find the factors of  $x^2 + 7x + 12$ ?

Q: Are there any rational functions that do not have vertical asymptotes?

Q: How would the graph of a rational function look if one of the factors were a factor of both the numerator and the denominator?

Part B:

Q: What happens to a fraction as the denominator approaches infinity?

Q: What does the degree of the numerator tell you about the horizontal asymptotes of these functions?

### Examples 3

Q: How can you determine which technique to use to find the horizontal asymptotes in this function?

Q: What strategies did you use to graph this function?

### Examples 4

Q: Is there a way to solve this problem if you do not have access to technology?

Q: What does the vertical asymptote mean in terms of the context?

### Examples 5

Q: Why do you check to see if the numerator of a rational function is equal to zero after finding the potential vertical asymptotes?

## Practice and Problem Solving

Complete MathXL for School: Practice and Problem Solve (online)

Complete MathXL for School: Enrichment (online)

Challenge: #12, 35 – key will be posted in Power School Learning.

## Lesson Quiz 4.2