Topic 3.6: Theorems about Roots of Polynomial Equations

Essential Question:

How are the roots of a polynomials equation related to the coefficients and degree of the polynomial?

Concept

CONCEPT The Rational Root Theorem

Let $P(x) = a_n x^n + a_{n-1} x^n + \ldots + a_1 x + a_0$ be a polynomial with integer coefficients.

If the polynomial equation $P(x) = 0$ has any rational roots, then each rational root is of the form $\frac{p}{q}$, where p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.

Concept

CONCEPT Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n solutions in the set of complex numbers.

If $P(x)$ has any factor of multiplicity m, count the solution associated with that factor m times. For example, the equation $(x - 3)^4 = 0$ has four solutions, each equal to 3.

Q: How might the Fundamental Theorem of Algebra be helpful when finding solutions to polynomial functions?

Notes:

CONCEPT Summary

Concept Summary Assess

CONCEPT SUMMARY Theorems About Roots of Polynomial Equations

	Words	Example
RATIONAL ROOT THEOREM	For the polynomial equation $0 = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x^1 + a_0$, there are a limited number of possible rational roots. Rational roots must have reduced form $\frac{p}{q}$ where p is an integer factor of a_0 and q is an integer factor of a_n. Use substitution or synthetic division to check roots.	$2x^3 + 3x^2 - 10x - 15 = 0$ $p = -15$; Factors of p: $\pm1, \pm3, \pm5, \pm15$ $q = 2$; Factors of q: $\pm1, \pm2$ Possible rational roots: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$ $-\frac{3}{2}$ is a root of the equation.
FUNDAMENTAL THEOREM OF ALGEBRA	If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n solutions in the set of complex numbers.	
CONJUGATE ROOT THEOREMS	**Irrational Conjugates**	**Complex Conjugates**
	Let P be a polynomial function with rational coefficients and let a and b be real numbers. Then if $a + \sqrt{b}$ is a root of $P(x) = 0$, then $a - \sqrt{b}$ is also a root of $P(x) = 0$.	Let P be a polynomial function with real coefficients and let a and b be real numbers. Then if $a + bi$ is a root of $P(x) = 0$, then $a - bi$ is also a root of $P(x) = 0$.

Examples & Questions

Examples 1
Q: How are the two polynomial functions similar? How are they different?

Examples 2
Q: Why does it appear that 4 is a zero of the function?
Q: How could you eliminate some of the possible roots that need to rest?

Examples 3
Q: Does the order in which you test the possible rational roots matter?

Q: How are synthetic division and the Quadratic Formula useful when finding all complex roots?

Examples 4
Part A
Q: What do you know about the product of a rational and an irrational number that is helpful when determining whether the coefficient is rational or irrational?

Part B
Q: How is the difference of two squares helpful in obtaining a rational coefficients?

Examples 5
Part A:
Q: What do you need to find to be able to write the equation with the given root?

Part B
Q: What is another way to determine that the function Q must have a degree of at least 4?

Practice and Problem Solving
Complete MathXL for School: Additional Practice (online)
Complete MathXL for School: Enrichment (online)

Challenge: #18, 24, 29, 35 – key will be posted in Power School Learning.

Lesson Quiz 3.6