Topic 3.3 Polynomial Identities

Essential Question:

How can you use polynomial identities to rewrite expressions efficiently?

Explore & Reason

Please complete online.

CONCEPT

Polynomial Identities

A mathematical statement that equates two polynomial expressions is an **identity** if one side can be transformed into the other side using mathematical operations. These polynomial identities are helpful tools used to multiply and factor polynomials.

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

Example: $25x^2 - 36y^2$

Substitute $5x$ for a and $6y$ for b.

$25x^2 - 36y^2 = (5x + 6y)(5x - 6y)$

Square of a Sum

$(a + b)^2 = a^2 + 2ab + b^2$

Example: $(3x + 4y)^2$

Substitute $3x$ for a and $4y$ for b.

$(3x + 4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$

$= 9x^2 + 24xy + 16y^2$

Difference of Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example: $8m^3 - 27$

Substitute $2m$ for a and 3 for b.

$8m^3 - 27 = (2m - 3)[(2m)^2 + (2m)(3) + 3^2]$

$= (2m - 3)(4m^2 + 6m + 9)$

Sum of Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example: $g^3 + 64h^3$

Substitute g for a and $4h$ for b.

$g^3 + 64h^3 = (g + 4h)[g^2 - (g)(4h) + (4h)^2]$

$= (g + 4h)(g^2 - 4gh + 16h^2)$

Examples & Questions

Examples 1

Q: When proving the identity, why does it make sense to start with the expression on the right side?

Examples 2

Part A:
Q: What do you recognize about the right side of the equation in the Square of a Sum Identity?

Part B:
Q: How do you determine the numerical values for *a* and *b*?

Examples 3

Part A:
Q: What does it mean for a term to be a perfect square?

Part B:
Q: How can you determine if a variable raised to a power is a perfect cube?

Part C:
Q: Why can you use the Sum of Cubes Identity when 11 and 5 are not divisible by 3?

Examples 4

Part A:
Q: How is he diagram in the example similar to Pascal's Triangle?

Q: What are the patterns of the powers when reading the terms from left to right?

Part B:
Q: What pattern do you notice in the coefficients of each row?

Examples 5

Part A:
Q: Why is it important to use Pascal's Triangle when applying the Binomial Theorem?

Part B:
Q: Wat do you think is a common mistake when using the Binomial Theorem to expand an expression with exponents in between the parenthesis?

Notes

CONCEPT Summary

Polynomial Identities

POLYNOMIAL IDENTITIES

Special polynomial identities can be used to multiply and factor polynomials.

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

Square of a Sum

$(a + b)^2 = a^2 + 2ab + b^2$

Difference of Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum of Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

BINOMIAL EXPANSION

The binomial expansion of $(a + b)^n$ has the following properties:

1) The expansion contains $n + 1$ terms.
2) The coefficients of each term are numbers from the nth row of Pascal's Triangle.
3) The exponent of a is n in the first term and decreases by 1 in each successive term.
4) The exponent of b is 0 in the first term and increases by 1 in each successive term.
5) The sum of the exponents in any term is n.

Row 0 — 1 — 1 — $(x + y)^0$

Row 1 — 1 1 — $1x + 1y$ — $(x + y)^1$

Row 2 — 1 2 1 — $1x^2 + 2xy + 1y^2$ — $(x + y)^2$

Row 3 — 1 3 3 1 — $1x^3 + 3x^2y + 3xy^2 + 1y^3$ — $(x + y)^3$

Row 4 — 1 4 6 4 1 — $1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$ — $(x + y)^4$

Practice and Problem Solving

Complete MathXL for School: Additional Practice (online)
Complete MathXL for School: Enrichment (online)

Challenge: #20, 64, 66, 69 – key will be posted in Power School Learning.

Lesson Quiz 3.3