Topic 2.4: Complex Numbers and Operations

Essential Question:

How can you represent and operate on numbers that are not on the real number line?

CONCEPT

Imaginary Number: $i=\sqrt{-1}$, $i^2=-1$

Complex Number: $a+bi$

Complex Conjugate: $(a+bi)\cdot(a-bi)=a^2+b^2$

CONCEPT Summary

Complex Numbers and Operations

The imaginary unit i is the number whose square is equal to -1: $\sqrt{-1}=i$, so $i^2=-1$.

Complex numbers are written in the form $a+bi$.

real numbers — imaginary unit

The four basic operations can be applied to complex numbers, such as $2+3i$ and $5-i$.

ADDITION

Add as you would with binomials with like terms.

$(2+3i)+(5-i)=7+2i$

SUBTRACTION

Subtract as you would with binomials with like terms.

$(2+3i)-(5-i)=-3+4i$

MULTIPLICATION

Distribute as you would with binomials.

$(2+3i)(5-i)=10-2i+15i-3i^2=13+13i$

DIVISION

Simplify so that the denominator is a real number. Multiply the numerator and denominator by the conjugate of the denominator.

$\frac{2+3i}{5-i}=\frac{(2+3i)(5+i)}{(5-i)(5+i)}=\frac{7+17i}{26}=\frac{7}{26}+\frac{17}{26}i$

Notes:

Examples & Questions

Examples 1

Q: How are the equations in Part A and Part B alike? How are they different?

Q: Why is the $\sqrt{-9}$ not equation to -3?

Examples 2

Q: In Part B, why is the difference written as an addition problem?
Q: How could the sum of two comples numbers be a real number?

Examples 3

Q: How is the problem in Part B like multiplying binomials?
Q:

Examples 4

Q: How does multiplying the denominator by its complex conjugate result in a real number?

Examples 5

Q: How is factoring the sum of squares different than factoring the difference of squares?
Q: Why is the first step in factoring the expression in Part B to factor out a 3?

Examples 6

Q: Graph $y = x^2 + 4$. How might be the graph suggest that there are no real solutions to the equation?
Q: Does the Zero Product Property hold for imaginary numbers?

Practice and Problem Solving

Complete MathXL for School: Enrichment (online)

Challenge: #12, 13, 46, 47, 50 – key will be posted in Power School Learning.

Lesson Quiz 2.4